

(Aim: 100/100 in Maths)

अभ्यास CLASS 10

ARITHMETIC PROGRESSION

Lecture 2

Abhi tak ki kahaani:-

AP:-



$$c.d.(d) = \text{अन्तरा-पिच्छमा}$$

AP की पहचान?

All c.d. are equal

→ n^{th} term of AP:-

$$a_n = a + (n-1)d$$

(CBSE)

LP : Find the number of natural numbers between 102 and 998 which are divisible by 2 and 5 both.

102 ————— 998

LCM: 10

LCM
आने end में 0 आए।

AP: 110, 120, 130, 140 990

Let n terms

$$a_n = 990$$

$$a + (n-1)d = 990$$

$$110 + (n-1)(10) = 990$$

$$(n-1)(10) = 990 - 110$$

$$(n-1)(10) = 880$$

$$n-1 = 88$$

$$n = 89$$

$\therefore$ 89 natural numbers div by 2 & 5 both

LP : Find the 10th term from the end of A.P. 4, 9, 14, ..., 254.

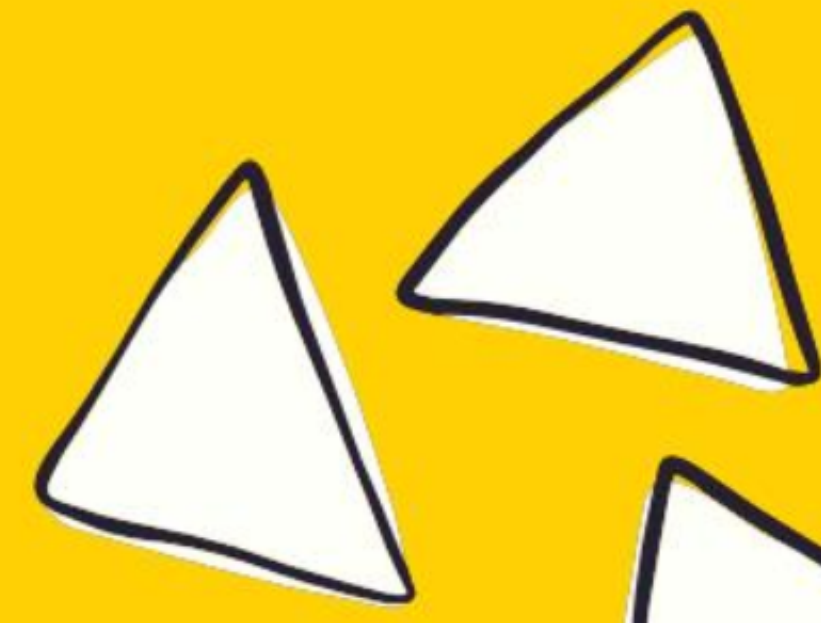
AP: 4, 9, 14, ..., 254
 $\xleftarrow{10^{\text{th}}}$

(Reverse) फलट पिरा : 254, ..., 14, 9, 4
 $\xrightarrow{10^{\text{th}}}$

$$\begin{aligned} r.d. &= 9 - 14 \\ d &= -5 \end{aligned}$$

$$\begin{aligned} a_{10} &= a + (10 - 1)d \\ &= 254 + 9(-5) \\ &= 254 - 45 \end{aligned}$$

$$a_{10} = 209$$



LP : Find the middle term of the A.P. 13 , 19 , ... , 247 .

13, 19, - - - - 247
n terms

$$a_n = 247$$

$$a + (n-1)d = 247$$

$$13 + (n-1)(6) = 247$$

$$(n-1)(6) = 234$$

$$n-1 = \frac{234}{6} = 39$$

$$n = 40$$

40 terms

$$\text{Middle term} \Rightarrow \left(\frac{40}{2}\right)^{\text{th}}, \left(\frac{40}{2} + 1\right)^{\text{th}}$$

$$\text{Middle terms} = 20^{\text{th}}, 21^{\text{st}}$$

20th 21st
↓ ↓
AP: 13, 19, $\square$, $\square$, - - - 247
40 terms

$$a_{20} = a + 19d$$

↓

$$a_{21} = a + 20d$$

↓

How to find middle term

$n = \text{odd}$

$$M = \left(\frac{n+1}{2} \right)^{\text{th}}$$

$$n = 7$$

$$M = \left(\frac{7+1}{2} \right)^{\text{th}} = \left(\frac{8}{2} \right)^{\text{th}} = 4^{\text{th}}$$

$n \rightarrow \text{even}$

$$M = \left(\frac{n}{2} \right)^{\text{th}}, \left[\frac{n}{2} + 1 \right]^{\text{th}}$$

$$n = 6$$
$$\left(\frac{6}{2} \right)^{\text{th}}, \left(\frac{6}{2} + 1 \right)^{\text{th}}$$

$$3^{\text{rd}}, 4^{\text{th}}$$

Sum of first n term of an AP:

AP: 2, 4, 6, 8 - - -
100 terms

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

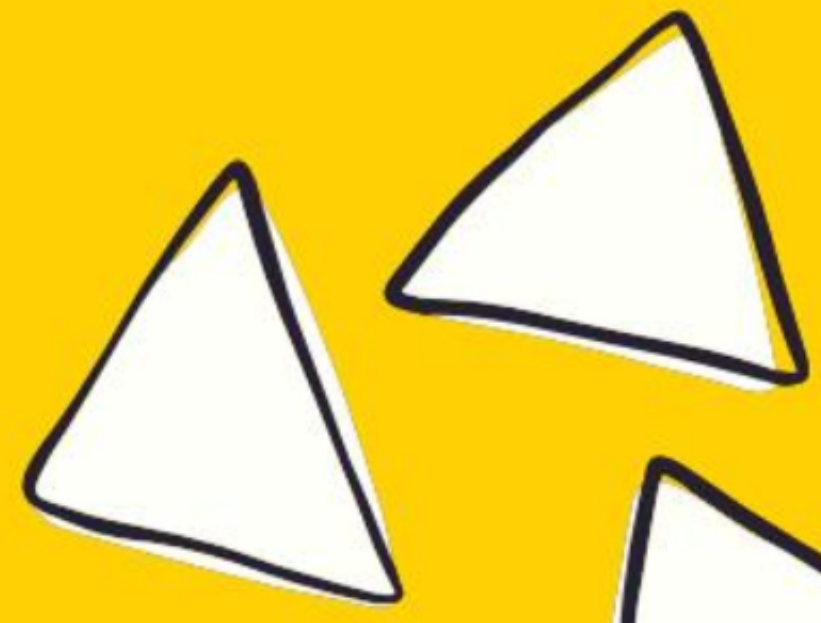
If you know
the value of
last term (l) / a_n

$$S_n = \frac{n}{2} [a + l]$$

a_n

अभय

$$\begin{aligned} S_{100} &= \frac{100}{2} [2(2) + (100-1)d] \\ &= 50 [4 + 99(2)] \\ &= 50 [4 + 198] \\ S_{100} &= 50 \times 202 \end{aligned}$$



#Q: Find the sum of first 50 terms of this AP:

3, 6, 9, - - - -

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{50} = \frac{50}{2} [2(3) + (50-1)(3)]$$

$$= 25 [6 + 49 \times 3]$$

$$= 25 [6 + 147]$$

$$S_{50} = 25 \times 153$$

LP : The sum of first six terms of an AP is 42. The ratio of its 10th term to its 30th term is 1:3. Calculate the first and thirteenth term of this AP.

ATQ $S_6 = 42$
 ~~$\frac{6}{2} [2a + (6-1)d] = 42$~~
 $2a + 5d = 14$ — (I)

$2d + 5d = 14$
 $7d = 14$
 $d = 2 = a$

ATQ $\frac{a_{10}}{a_{30}} = \frac{1}{3}$

$\frac{a + 9d}{a + 29d} = \frac{1}{3}$

$3a + 27d = a + 29d$

$3a - a = 29d - 27d$

$2a = 2d$

$a = d$

$a_1 = a = 2$

$a_{13} = a + (13-1)d$
 $= 2 + 12(2)$
 $= 2 + 24 = 26 = a_{13}$

LP : In an AP, the first term is 2, the last term is 29 and the sum of the terms is 155. Find the common difference of the AP.

$a_1, \dots, a_n$ $a_n = 29$

$a = 2$, $l = 29$, $S_n = 155$
 let 'n' terms are there

$d = ?$

$9d = 27$
 $d = \frac{27}{9} = 3$

$d = 3$

$31n = 310$

$n = \frac{310}{31}$

$n = 10$

$S_n = 155$

$\frac{n}{2} [a + l] = 155$

$\frac{n}{2} [2 + 29] = 155$

$n(31) = 155 \times 2$

$a_n = 29$

$a + (n-1)d = 29$

$2 + (10-1)d = 29$

$9d = 29 - 2$



*k⁵B ⇒ a_n देकर S_n पूछ लें।

अभ्यास

LP : If the n th term of an AP is $(2n+1)$, find the sum of first n terms of AP.

$$a_n = (2n+1)$$

$$n=1 - a_1 = a = (2(1)+1) \Rightarrow \textcircled{3} = a$$

$$a_2 = 2(2)+1 \Rightarrow \textcircled{5}$$

$$a_3 = 2(3)+1 \Rightarrow \textcircled{7}$$

AP : 3, 5, 7, ...

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$= \frac{n}{2} (2(3) + (n-1)(2))$$

$$= \frac{n}{2} (6 + (n-1)2)$$

$$= \frac{n}{2} (6 + 2n - 2)$$

$$= \frac{n}{2} (4 + 2n)$$

$$= \frac{n}{2} \times 2(2+n) = \textcircled{n(n+2)} = S_n$$

#BOARD KE SAWAAL

अभ्यास

LP : If $S_n = 3n^2 - 4n$, find the n th term of AP. [CBSE 2019]

$$S_n = 3n^2 - 4n$$

$$n=1 \rightarrow S_1 = a_1 = 3(1)^2 - 4(1) \\ = 3 - 4 = -1 = a_1$$

$$n=2 \rightarrow S_2 = a_1 + a_2$$

$$\Rightarrow 3(2)^2 - 4(2) = a_1 + a_2$$

$$\Rightarrow 3(4) - 8 = a_1 + a_2$$

$$\Rightarrow 12 - 8 = -1 + a_2$$

$$\Rightarrow 4 + 1 = a_2 \Rightarrow a_2 = 5$$

S_n देकार a_n पृथक् लिया।

AP: $-1, 5, \dots$

$$d = 5 - (-1) \\ = 5 + 1 = 6$$

$$a_n = a + (n-1)d$$

$$= -1 + (n-1)(6)$$

$$= -1 + 6n - 6$$

$$a_n = 6n - 7$$

#LP: Solve $1+4+7+10+\dots+x = 287$ [CBSE 2020]

AP : $1+4+7+10+\dots+x = 287$ $\leftarrow a_n$

let n terms

$S_n = 287$

$a_n = x$

$\frac{n}{2}(a+l) = 287$

$\frac{n}{2}(1+x) = 287$

$n(1+x) = 287 \times 2$

$n(1+x) = 574$

$n = \frac{574}{(1+x)}$

$a + (n-1)d = x$
 $1 + (n-1)(3) = x$

$1 + \left(\frac{574}{1+x} - 1\right)(3) = x$

$\Rightarrow \left(\frac{574 - (1+x)}{1+x}\right)(3) = x - 1$

$\Rightarrow \left(\frac{574 - 1 - x}{1+x}\right) 3 = x - 1$

$= (573 - x)3 = (x-1)(x+1)$

$\frac{573}{1719}$

$1719 - 3x = x^2 - 1$

$x^2 + 3x - 1720 = 0$

$x = 40$



#LP: Ramkali required Rs 2500 after 12 weeks to send her daughter to school. She saved Rs 100 in the first week and increased her weekly saving by Rs 20 every week. Find whether she will be able to send her daughter to school after 12 weeks or not. What value is generated in the above situation? [CBSE 2015]

$$\begin{array}{ccc} 1^{st} & 2^{nd} & 3^{rd} \\ \downarrow & \downarrow & \downarrow \\ 100 & + 120 & + 140 - - - - - \end{array} = \text{Rs } \underline{2500}$$

12 weeks

$$S_{12} = \frac{12}{2} [2(100) + (12-1)20]$$

$$= 6 [200 + 11 \times 20]$$

$$= 6 [200 + 220]$$

$$\Rightarrow 6 [420] = \text{Rs } \underline{2520}$$

$$\begin{array}{r} 420 \\ \times 6 \\ \hline 2520 \end{array}$$

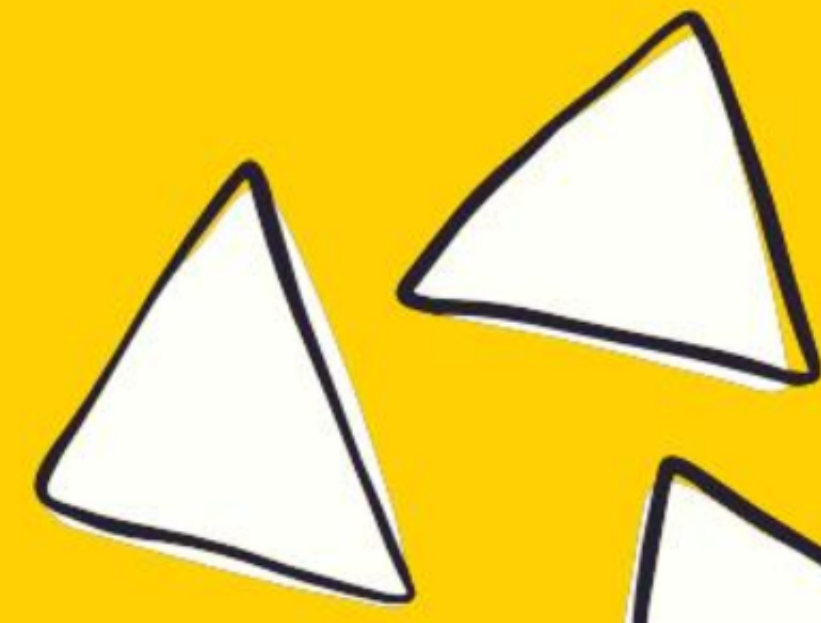
#LP: Find the sum: $(4 - \frac{1}{n}) + (4 - \frac{2}{n}) + (4 - \frac{3}{n}) + \dots$ upto n terms

[CBSE 2017]

$$(4 - \frac{1}{n}), (4 - \frac{2}{n}), (4 - \frac{3}{n}) \dots n \text{ terms.}$$

$$\begin{aligned} d &= \text{अवसा - पि पदमा} \\ &= (4 - \frac{2}{n}) - (4 - \frac{1}{n}) \\ &= 4 - \frac{2}{n} - 4 + \frac{1}{n} \\ &= -\frac{2}{n} + \frac{1}{n} \\ &= \frac{-2+1}{n} = \frac{-1}{n} = -\frac{1}{n} \end{aligned}$$

$$\begin{aligned} S_n &= \frac{n}{2} (2a + (n-1)d) \\ &= \frac{n}{2} (2(4 - \frac{1}{n}) + (n-1)(-\frac{1}{n})) \\ &= \frac{n}{2} [8 - \frac{2}{n} - 1 + \frac{1}{n}] \\ &= \frac{n}{2} [7 - \frac{1}{n}] \\ &= \frac{n}{2} \cdot \frac{7n-1}{n} = \frac{7n-1}{2} \end{aligned}$$



* #LP: If the p^{th} term of an AP is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, prove that the sum of first pq terms of the AP is $(pq+1)/2$. [CBSE 2017]

$$a_p = \frac{1}{q} \Rightarrow a + (p-1)d = \frac{1}{q} \quad \text{--- (I)}$$

$$a_q = \frac{1}{p} \Rightarrow a + (q-1)d = \frac{1}{p} \quad \text{--- (II)}$$

$$(p-1)d - (q-1)d = \frac{1}{q} - \frac{1}{p}$$

$$d[p-1-q+1] = \frac{p-q}{pq}$$

$$d(\cancel{p}-\cancel{q}) = \frac{p-q}{pq}$$

$$\boxed{d = \frac{1}{pq}}$$

$$\text{put } d = \frac{1}{pq} \text{ in (I)}$$

$$a + (p-1)\left(\frac{1}{pq}\right) = \frac{1}{q}$$

$$a + \frac{p-1}{pq} = \frac{1}{q}$$

$$a = \frac{1}{q} - \frac{p-1}{pq}$$

$$a = \frac{p - p + 1}{pq}$$

$$\boxed{a = \frac{1}{pq}}$$

$$S_{pq} = \frac{pq+1}{2}$$

$$a = -\frac{1}{p_2}, d = \frac{1}{p_2}$$

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$S_{p_2} = \frac{p_2}{2} [2a + (p_2-1)d]$$

$$= \frac{p_2}{2} \left[\frac{2}{p_2} + (p_2-1) \left(\frac{1}{p_2} \right) \right]$$

$$= \frac{p_2}{2} \left[\frac{2 + p_2 - 1}{p_2} \right]$$

$$S_{p_2} = \frac{p_2 + 1}{2}$$

आभार

THANK YOU

COODIES 🥰