[Aim: 100/100 in Maths]

TRIGONOMETRY

$\underbrace{\text{TRI}}_{3}\ \underbrace{\text{GON}}_{\text{sides}}\ \underbrace{\text{OMETRY}}_{\text{measurement}}$

$\underbrace{3 \quad \text{sides}}$ → Right △ Triangle

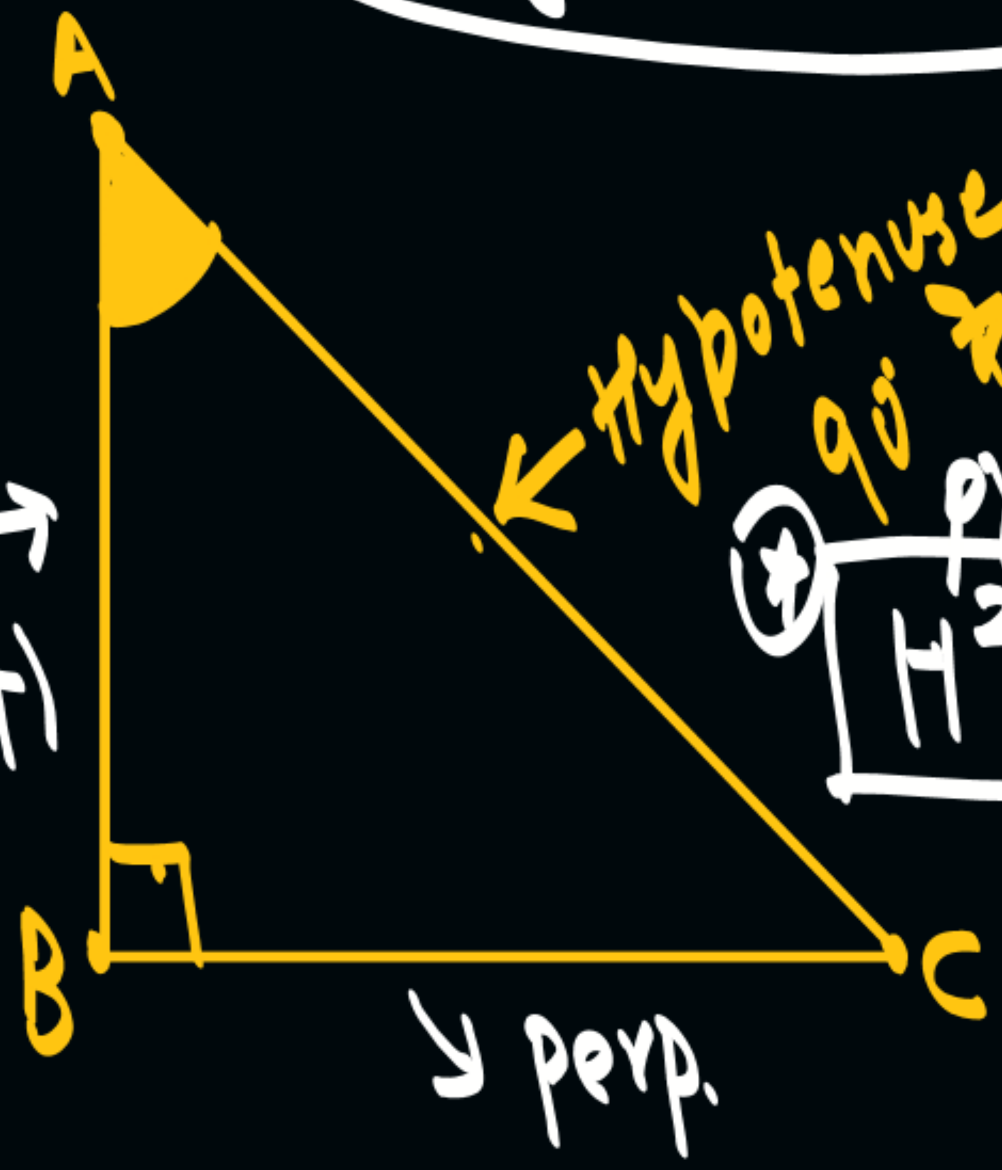
Main focus → Right △ Triangle

→ **Basic Lecture**

→ **V. Imp** (Identities)

Base → (खाल वाली)

← Hypotenuse (fix) 90° के सामने वाला

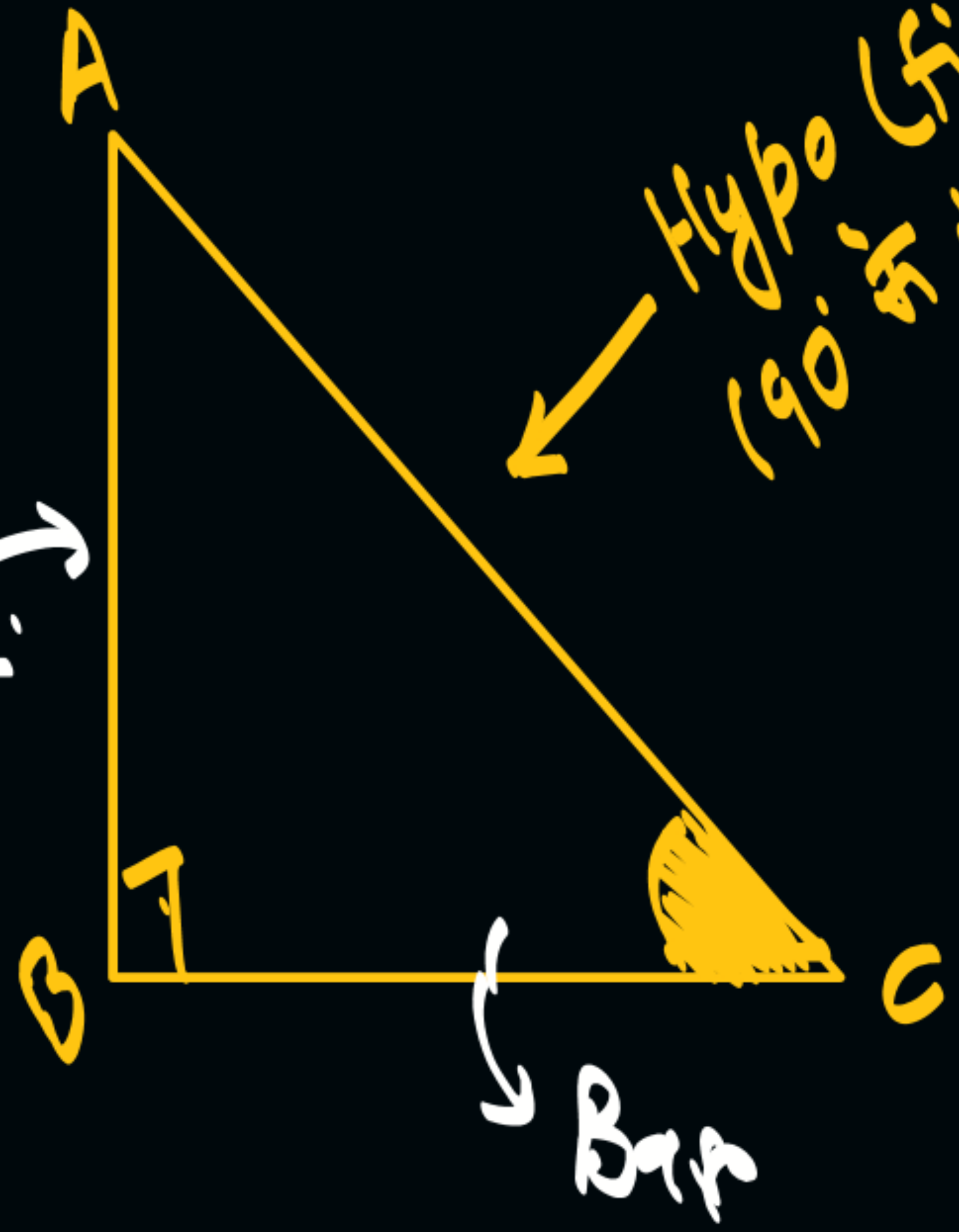


★ pythagorus

$$\boxed{H^2 = P^2 + B^2}$$

Perp. →

Hypo (fix) (90° के सामने)

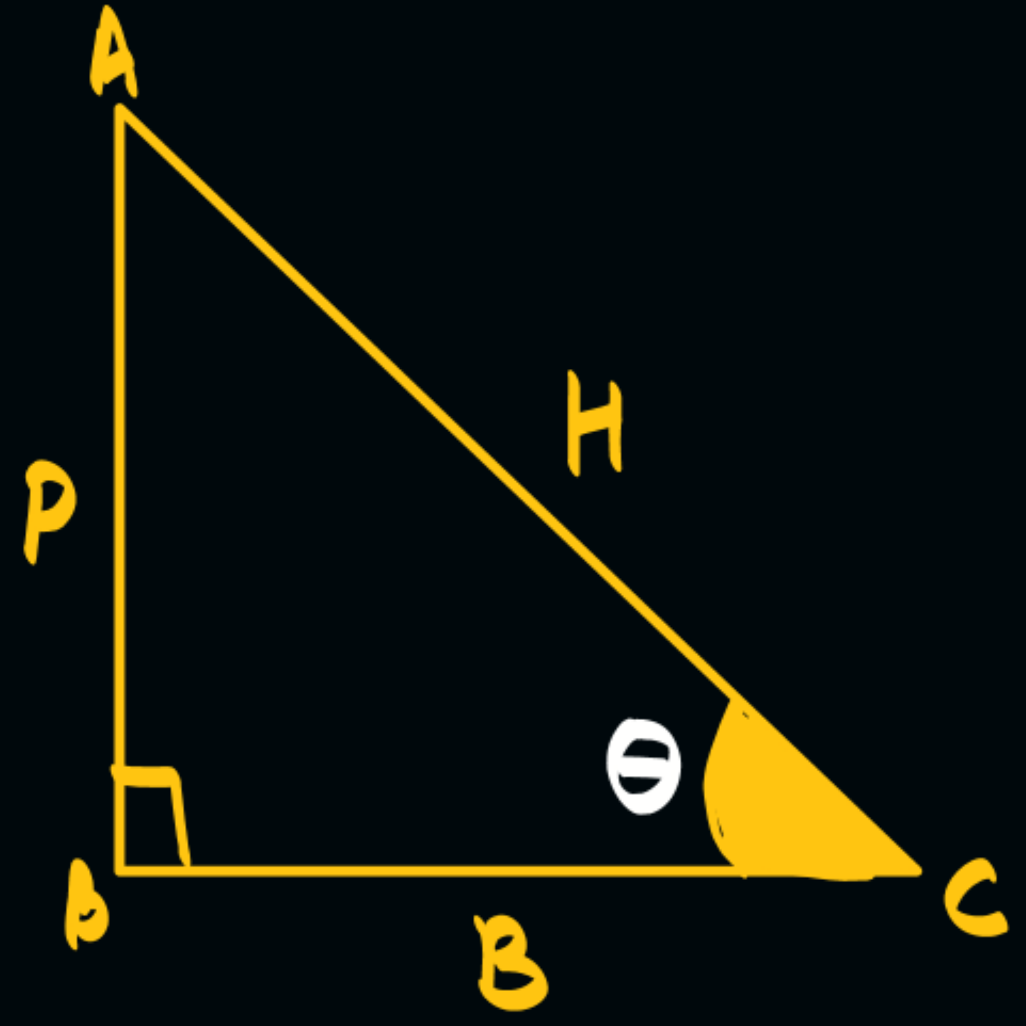


→ Trigometric Ratios:-

P, H, B

$\sin\theta = \frac{P}{H}$ (sin of angle θ) | $\cos\theta = \frac{B}{H}$ | $\tan\theta = \frac{P}{B}$

$\operatorname{cosec}\theta = \frac{H}{P}$ | $\sec\theta = \frac{H}{B}$ | $\cot\theta = \frac{B}{P}$



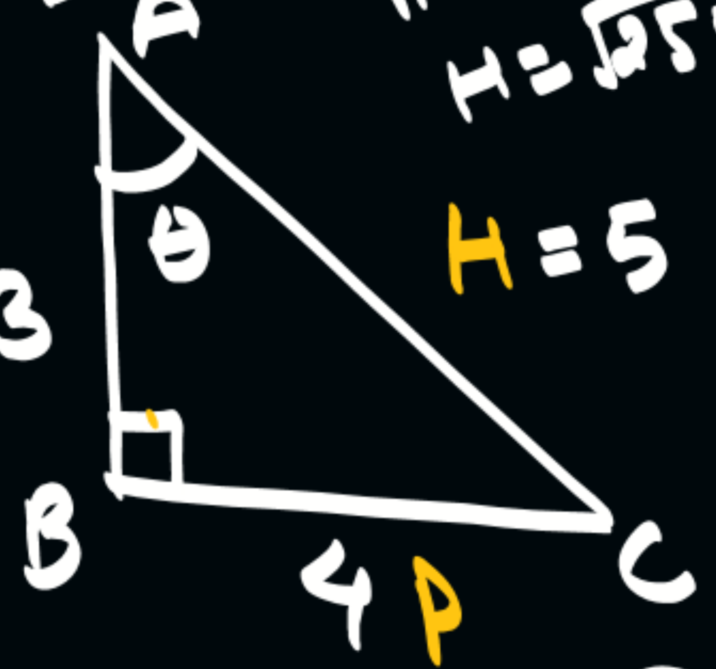
Pytha

$H^2 = P^2 + B^2$

$H^2 = 16 + 9$

$H = \sqrt{25} = 5$

Q



$\cos\theta = \frac{B}{H} = \frac{3}{5}$

$\tan\theta = \frac{P}{B} = \frac{4}{3}$

K3B Trick to remember T-ratios:-

$\sin\theta$	$\cos\theta$	$\tan\theta$
Papa	Bidi	Piyoge
Ha	Ha	Beta
$\operatorname{cosec}\theta$	$\sec\theta$	$\cot\theta$

$\operatorname{cosec}\theta \longleftrightarrow \sin\theta$ (Reciprocal) $\left[\sin\theta = \dfrac{1}{\operatorname{cosec}\theta} \;\middle|\; \operatorname{cosec}\theta = \dfrac{1}{\sin\theta}\right]$

$\cos\theta \leftrightarrow \sec\theta$ $\left[\cos\theta = \dfrac{1}{\sec\theta} \;\middle|\; \sec\theta = \dfrac{1}{\cos\theta}\right]$

$\tan\theta \leftrightarrow \cot\theta$ $[\quad]$

LP: If $\sin A = \frac{4}{5}$, find $\cos A$ & $\tan A$.

$$\sin A = \frac{4}{5}$$

$$\frac{P}{H} = \frac{4}{5}$$

$$P = 4k, \quad H = 5k$$

Pytha, $H^2 = P^2 + B^2$

$$(5k)^2 = (4k)^2 + B^2$$

$$25k^2 - 16k^2 = B^2$$

$$9k^2 = B^2$$

$$\sqrt{9k^2} = B \rightarrow \boxed{B = 3k}$$

Now, $P = 4k$, $H = 5k$, $B = 3k$

$$\cos A = \frac{B}{H} = \frac{3k}{5\cancel{k}} = \frac{3}{5}$$

$$\tan A = \frac{P}{B} = \frac{4\cancel{k}}{3\cancel{k}} = \frac{4}{3}$$

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cosec A, sec A, cot A ??

$$\operatorname{cosec} A = \frac{H}{P} \,/\, \frac{1}{\sin A} = \frac{5}{4}$$

$$\sec A \rightarrow \frac{5}{3}$$

$$\cot A \rightarrow \frac{3}{4}$$

#LP: In a right triangle ABC, right-angled at B, if tan A = 1, then verify that 2 sin A cos A = 1.

$\tan A = 1$

$\frac{P}{B} = \frac{1}{1}$

$P = 1k, \; B = 1k$

$H^2 = P^2 + B^2$

$= (1k)^2 + (1k)^2$

$H^2 = 2k^2$

$H = \sqrt{2k^2}$

$H = \sqrt{2}\,k$

Verify: $2 \underbrace{\sin A} \cdot \underbrace{\cos A} = 1$

LHS $2 \times \frac{P}{H} \times \frac{B}{H}$

$\Rightarrow 2 \times \frac{1k}{\sqrt{2}k} \times \frac{1k}{\sqrt{2}k}$

$\Rightarrow 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

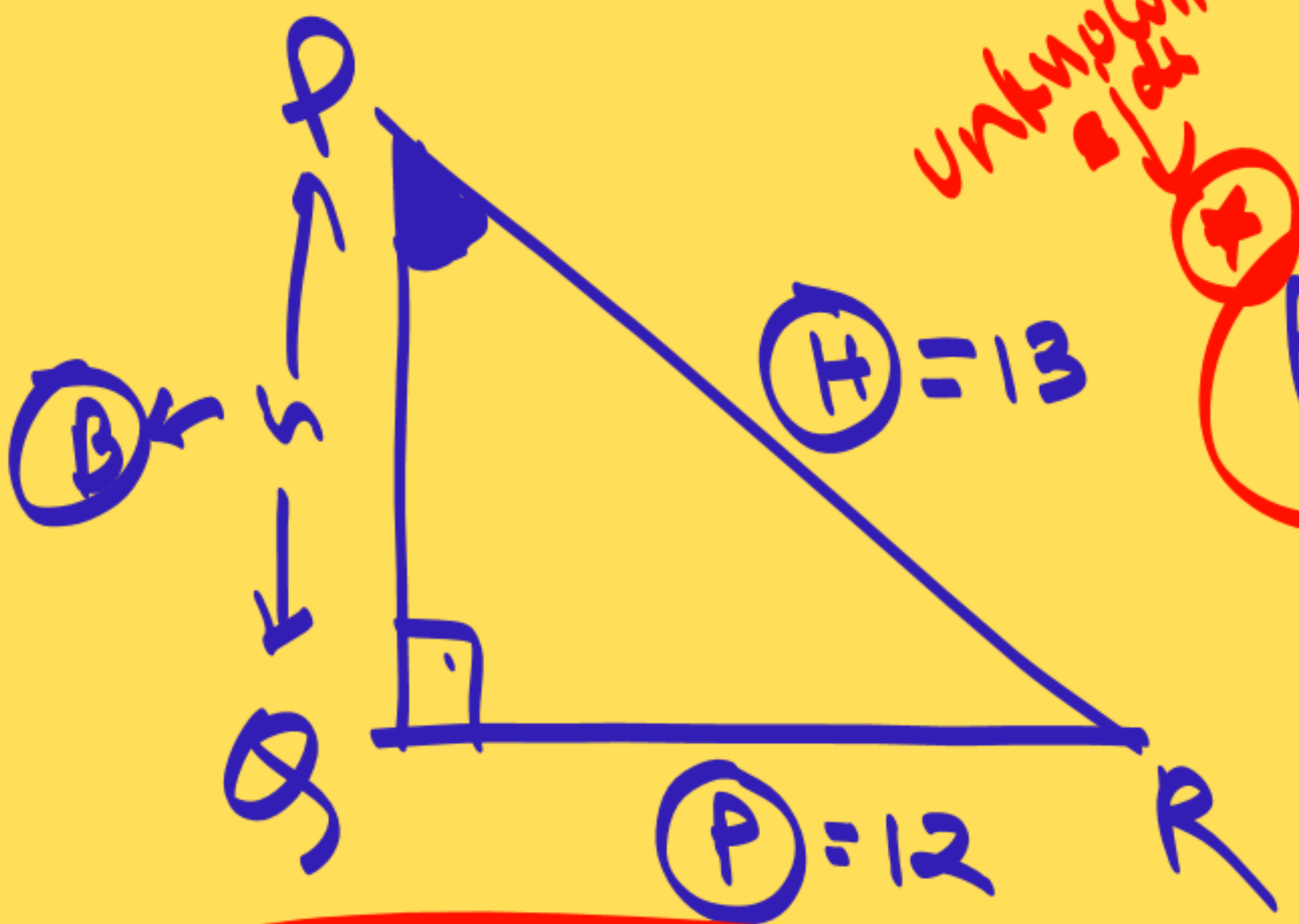
$\Rightarrow \cancel{2}^{1} \times \frac{1}{\cancel{2}} \Rightarrow 1$

RHS

$\therefore$ LHS = RHS

HP

#LP. In Triangle PQR, right-angle at Q. PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, and tan P.



unknown side

Pytha.

$PR + QR = 25$, $PQ = 5$ cm

Base

$H^2 = P^2 + B^2$

$PR^2 = QR^2 + (5)^2$

$PR^2 - QR^2 = 25$

$(PR - QR)(PR + QR) = 25$

$(PR - QR)(\cancel{25}) = (\cancel{25})$ 1

$PR - QR = 1$

$PR = 1 + QR = 1 + 12 = 13 = PR$

$(1 + QR) + QR = 25$

$1 + 2QR = 25$

$2QR = 24$

$QR = 12$

sin P =

tan P =

#TABLE

$\theta \rightarrow$	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D.
$\operatorname{cosec}\theta$	N.D.	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	N.D
$\cot\theta$	N.D.	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

#LP:

$$\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$$

(A) sin 60° (B) cos 60° (C) tan 60° D) sin 30°

$$\frac{2\tan 30}{1+\tan^2 30} \Rightarrow \frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} \Rightarrow \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}$$

$$\Rightarrow \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$\Rightarrow \frac{\cancel{2}}{\sqrt{\cancel{3}}}\times\frac{\cancel{3}^{\sqrt{3}}}{\cancel{4}_{2}} \Rightarrow \frac{\sqrt{3}}{2} \rightarrow \sin 60$$

$\tan^2 30$

$(\tan 30)^2$

#LP:

$$\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ} = \frac{5 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

concept :-

$\sin A = \sin B$

$\Rightarrow \angle A = \angle B$

$\xrightarrow{\text{converse}}$

if $\angle A = \angle B$

$\sin A = \sin B$

→ Same for all trignometric Ratio.

$\cancel{\sin A = \sin B}$

$\cancel{A = B}$

LP :-

$\sin A = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$, find $\angle A$ = ?

$$\sin A = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$\sin A = \frac{1}{2}$$

$$\sin A = \sin 30$$

$$A = 30^\circ$$

#LP: If $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0° < \theta < 90°$, And the value of θ

$$\sqrt{3}\sin\theta - \cos\theta = 0$$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\sqrt{3} = \frac{\cos\theta}{\sin\theta}$$

$$\sqrt{3} = \cot\theta$$

$$\cot 30 = \cot\theta$$

$$\boxed{30° = \theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

#LP: Find the value of x in the following:
tan 3x = sin 45° cos 45° + sin 30°

$$\tan(3x) = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2}$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$\tan(3x) = 1$$

$$\tan(3x) = \tan 45^\circ$$

$$3x = 45$$

$$x = \frac{45}{3} = 15 = 2.$$

#LP: If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$; $0° < A+B \leq 90°$; $A > B$, find A and B.

$\tan(A+B) = \sqrt{3}$

$\tan(A+B) = \tan 60$

$\boxed{A+B = 60}$

$\tan(A-B) = \frac{1}{\sqrt{3}}$

$\tan(A-B) = \tan 30$

$\boxed{A-B = 30}$

$A = 45$

$B = 15$

HW

~~#11~~ . In an acute angled triangle ABC, if $\tan(A + B - C) = 1$ and $\sec(B + C - A) = 2$, find the value of A, B and C.

#LP: State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$.

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as θ increases.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

(v) $\cot A$ is not defined for $A = \theta°$

#LP: i) The value of tan A is always less than 1.

(ii) sec A = 12/5 for some value of angle A.

(iii)cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) sin θ = 4/5 for some angle θ.

#LP: tan = 4/5 then the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}$

(A) 5/7

(B) 4/7

(C) 9/7

(D) 1/7

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{4}{5}$

$\frac{\sin\theta}{\cos\theta} = \frac{4}{5}$

$\Rightarrow \frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}$

divide Num & Den by $\cos\theta$

$\Rightarrow \frac{5\frac{\sin\theta}{\cos\theta} - \frac{3\cancel{\cos\theta}}{\cancel{\cos\theta}}}{5\frac{\sin\theta}{\cos\theta} + \frac{3\cancel{\cos\theta}}{\cancel{\cos\theta}}} \Rightarrow \frac{5\tan\theta - 3}{5\tan\theta + 3}$

$\Rightarrow \frac{\cancel{5}\times\frac{4}{\cancel{5}} - 3}{\cancel{5}\times\frac{4}{\cancel{5}} + 3} \Rightarrow \frac{1}{7}$

#LP: If mcot A= n, find the value of $\dfrac{m\sin A - n\cos A}{n\sin A + m\cos A}$

$m\cot A = n$

$\cot A = \dfrac{n}{m}$

$\dfrac{\cos A}{\sin A} = \dfrac{n}{m}$

$\Rightarrow \dfrac{m\sin A - n\cos A}{n\sin A + m\cos A}$

div num & den by sin A

$\Rightarrow \dfrac{\dfrac{m\cancel{\sin A}}{\cancel{\sin A}} - \dfrac{n\cos A}{\sin A}}{\dfrac{n\cancel{\sin A}}{\cancel{\sin A}} + \dfrac{m\cos A}{\sin A}} \Rightarrow \dfrac{m - n\cot A}{n + m\cot A}$

$\Rightarrow \dfrac{m - n\cdot\frac{n}{m}}{n + \cancel{m}\,\frac{n}{\cancel{m}}}$

$\Rightarrow \dfrac{m - \frac{n^2}{m}}{2n}$