[Aim: 100|100 in Maths]

अभय CLASS 10

TRIGONOMETRY

Basics

Identities

L - 2

sin	cos	tan
P	B	P
H	H	B
cosec	sec	cot

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

$\tan\theta = \frac{1}{\cot\theta} = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

	0	30	45	60	90
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D
$\operatorname{cosec}\theta$	N.D	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	N.D
$\cot\theta$	N.D	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

TRIGONOMETRIC IDENTIES

- $\sin^2\theta + \cos^2\theta = 1$
- $\sec^2\theta - \tan^2\theta = 1$
- $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$\sec^2\theta = 1 + \tan^2\theta$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

LP:

$\operatorname{cosec}^2\theta \cdot (1-\cos^2\theta) = k$, find k.

(A) 2

(B) 3

(C) 1

(D) 0

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$$\Rightarrow \operatorname{cosec}^2\theta\ \underbrace{(1-\cos^2\theta)}$$

$$\Rightarrow \frac{1}{\sin^2\cancel{\theta}} \times \sin^2\cancel{\theta} \Rightarrow 1$$

#11: $\sin^2\theta + \sin\theta = 1$, find value of $\cos^2\theta + \cos^4\theta$

(A) -1

(B) 1 ✓

(C) 0

(D) 2

$$\boxed{\begin{aligned} &\sin^2\theta + \cos^2\theta = 1 \\ &\sin^2\theta = 1 - \cos^2\theta \ ✓ \end{aligned}}$$

$$\cancel{1} - \cos^2\theta + \sin\theta = \cancel{1}$$

$$\boxed{\cos^2\theta = \sin\theta}$$

$$\sin\theta + (\cos^2\theta)^2$$

$$\Rightarrow \sin\theta + (\sin\theta)^2$$

$$\Rightarrow \sin\theta + \sin^2\theta$$

$$\Rightarrow 1 \ ✓$$

LP : Prove that : $\dfrac{\text{Sin A} - 2\,\text{Sin}^3\text{A}}{2\,\text{Cos}^3\text{A} - \text{Cos A}} = \text{Tan A}$

LHS

$$\frac{\sin A\,(1 - 2\sin^2 A)}{\cos A\,(2\cos^2 A - 1)}$$

$$\Rightarrow \tan A \,\frac{(1 - 2\sin^2 A)}{(2\cos^2 A - 1)}$$

$$\Rightarrow \tan A \,\frac{(1 - 2(1 - \cos^2 A))}{(2\cos^2 A - 1)}$$

$\left[\because \sin^2 A + \cos^2 A = 1,\quad \sin^2 A = 1 - \cos^2 A\right]$

$$\Rightarrow \tan A \,\frac{(1 - 2 + 2\cos^2 A)}{(2\cos^2 A - 1)}$$

$$\Rightarrow \tan A \,\frac{(-1 + 2\cos^2 A)}{(2\cos^2 A - 1)}$$

$$\Rightarrow \tan A \,\frac{\cancel{(2\cos^2 A - 1)}}{\cancel{(2\cos^2 A - 1)}}$$

$$\Rightarrow \boxed{\tan A} = \text{RHS}$$

HP

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LP : If $\sin\theta - \cos\theta = 0$, then find the value of $\sin^4\theta + \cos^4\theta$.

$\sin\theta = \cos\theta$

$\boxed{\theta = 45^\circ}$

$\boxed{\sqrt{a} = a^{\frac{1}{2}}}$

$\Rightarrow (\sin 45)^4 + (\cos 45)^4$

$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$

$\Rightarrow 2\left(\frac{1}{\sqrt{2}}\right)^4$

$\Rightarrow 2\,\frac{(1)^4}{(2^{\frac{1}{2}})^{4}} \Rightarrow \cancel{2} \times \frac{1}{2^{\cancel{2}}} \Rightarrow \frac{1}{2}$

LP : If tan a + cot a = 2 , then tan a^{20} + cot a^{20} =

A. 0
B. 2
C. 20
D. 2^{20}

$\tan\alpha + \dfrac{1}{\tan\alpha} = 2$

$\Rightarrow \dfrac{\tan^2\alpha + 1}{\tan\alpha} = 2$

$\Rightarrow \tan^2\alpha + 1 = 2\tan\alpha$

Quadratic $\boxed{\tan^2\alpha - 2\tan\alpha + 1 = 0}$

$\Rightarrow \tan^2\alpha - \tan\alpha - \tan\alpha + 1 = 0$

$\Rightarrow \tan\alpha(\tan\alpha - 1) - 1(\tan\alpha - 1) = 0$

$\Rightarrow (\tan\alpha - 1)(\tan\alpha - 1) = 0$

$\Rightarrow \tan\alpha - 1 = 0$

$\tan\alpha = 1$

$\cot\alpha = \dfrac{1}{\tan\alpha} = \dfrac{1}{1}$

$\therefore \boxed{\cot\alpha = 1}$

$(\tan\alpha)^{20} + (\cot\alpha)^{20}$

$(1)^{20} + (1)^{20}$

$1 + 1 \Rightarrow 2$

#K-5

$\sec^2\theta - \tan^2\theta = 1$

$a^2 - b^2 = (a+b)(a-b)$

$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$$(\sec\theta - \tan\theta) = \frac{1}{(\sec\theta + \tan\theta)}$$ → Reciprocal.

→ Can we say the same for $\operatorname{cosec}\theta + \cot\theta$??

we know, $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$a^2 - b^2$

$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

$$\boxed{\operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}}$$

→ can we say same thing for $\sin\theta + \cos\theta$??

$\sin^2\theta + \cos^2\theta = 1$

$a^2 + b^2$

#LP: If $\sec\theta + \tan\theta = 2$

find $\sec\theta - \tan\theta$? !

reciprocals of each other.

$$\sec\theta - \tan\theta = \frac{1}{2}$$

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LP : If $\sec\theta + \tan\theta = p$, then $\tan\theta$ is

A. $p^2 + 1 / 2p$

B. $p^2 - 1 / 2p$

C. $p^2 - 1 / p^2 + 1$

D. $p^2 + 1 / p^2 - 1$

$$\cancel{\sec\theta} - \tan\theta = \frac{1}{p}$$

$$\cancel{\sec\theta} + \tan\theta = p$$

$$-2\tan\theta = \frac{1}{p} - p$$

$$2\tan\theta = p - \frac{1}{p}$$

$$2\tan\theta = \frac{p^2-1}{p}$$

$$\tan\theta = \frac{p^2-1}{2p}$$

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LP : Prove the identity : $\frac{1}{\operatorname{cosec}\theta + \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta - \cot\theta}$

LHS $(\cancel{\operatorname{cosec}}\theta - \cot\theta) - \cancel{\operatorname{cosec}}\theta$

$\Rightarrow -\cot\theta$

RHS

$\rightarrow \operatorname{cosec}\theta - (\operatorname{cosec}\theta + \cot\theta)$

$\rightarrow \cancel{\operatorname{cosec}\theta} - \cancel{\operatorname{cosec}\theta} - \cot\theta$

$\rightarrow -\cot\theta$

LHS = RHS

HP

$K^3B \Rightarrow$ addition given है ⟶ multiplication पूछा है

LP : If $\underbrace{\sin\theta + \cos\theta = \sqrt{3}}$, then find the value of $\underbrace{\sin\theta \cdot \cos\theta}$

squaring both sides

sq. both sides

$$\Rightarrow (\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$$

$$\Rightarrow \underbrace{\sin^2\theta + \cos^2\theta} + 2\sin\theta\cos\theta = 3$$

$$1 + 2\sin\theta\cos\theta = 3$$

$$\cancel{2}\sin\theta\cos\theta = \cancel{2}$$

$$\sin\theta \cdot \cos\theta = 1$$

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LP : If cos θ + sin θ = √2 cos θ , show that cos θ - sin θ = √2 sin θ

$$\sin\theta = \sqrt{2}\cos\theta - \cos\theta$$

$$\sin\theta = \cos\theta(\sqrt{2}-1)$$

$$\frac{\sin\theta}{\sqrt{2}-1} = \cos\theta$$

rationalise

$$\Rightarrow \frac{\sin\theta}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \cos\theta$$

$$\Rightarrow \frac{\sin\theta(\sqrt{2}+1)}{(\sqrt{2})^2-(1)^2} = \cos\theta$$

$$\Rightarrow \sin\theta(\sqrt{2}+1) = \cos\theta$$

2 - 1 → 1

LHS $\cos\theta - \sin\theta$

sin में convert करेगा

$$\sin\theta(\sqrt{2}+1) - \sin\theta$$

$$\sqrt{2}\sin\theta + \cancel{\sin\theta} - \cancel{\sin\theta}$$

$$\Rightarrow \sqrt{2}\sin\theta = \text{RHS}$$

HP

LP : If $x = p\sec\theta + q\tan\theta$ and $y = p\tan\theta + q\sec\theta$, then prove that $x^2 - y^2 = p^2 - q^2$

$x^2 = (p\sec\theta + q\tan\theta)^2$

$y^2 = (p\tan\theta + q\sec\theta)^2$

To prove : $x^2 - y^2 = p^2 - q^2$

LHS $x^2 - y^2$

$(p\sec\theta + q\tan\theta)^2 - (p\tan\theta + q\sec\theta)^2$

$(a+b)^2$

$\Rightarrow p^2\sec^2\theta + q^2\tan^2\theta + 2(p\sec\theta)(q\tan\theta) - [p^2\tan^2\theta + q^2\sec^2\theta + 2(p\tan\theta)(q\sec\theta)]$

$\Rightarrow p^2\sec^2\theta + q^2\tan^2\theta + \cancel{2pq\sec\theta\tan\theta} - p^2\tan^2\theta - q^2\sec^2\theta - \cancel{2pq\sec\theta\tan\theta}$

$\Rightarrow p^2(\sec^2\theta - \tan^2\theta) + q^2(\tan^2\theta - \sec^2\theta)$

$\Rightarrow p^2(1) - q^2(\underbrace{\sec^2\theta - \tan^2\theta}_{1}) \Rightarrow \boxed{p^2 - q^2}$ RHS

HP

LP : Prove that : $\sqrt{1 + \sin A / 1 - \sin A} = \sec A + \tan A$

$\sin^2 A + \cos^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$

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To prove ⇒ $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Rationalisation

LHS

$\sqrt{\dfrac{1+\sin A}{1-\sin A} \times \dfrac{1+\sin A}{1+\sin A}}$

$\Rightarrow \sqrt{\dfrac{(1+\sin A)^2}{1-\sin^2 A}} \Rightarrow \sqrt{\dfrac{(1+\sin A)^2}{(\cos A)^2}}$

$\cancel{\sqrt{}}\left(\dfrac{1+\sin A}{\cos A}\right)^{\cancel{2}}$

$\Rightarrow \dfrac{1+\sin A}{\cos A}$

$\Rightarrow \dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}$

$\Rightarrow \sec A + \tan A = RHS$

HP

LP : If $1 + \sin^2\theta = 3 \sin\theta \cos\theta$ then prove that $\tan\theta = 1$ or $\tan\theta = \frac{1}{2}$

LP : If $\sin A + \sin^3 A = \cos^2 A$, prove that $\cos^6 A - 4\cos^4 A + 8\cos^2 A = 4$

LP : Prove that : $\dfrac{\tan^3\theta}{1+\tan^2\theta} + \dfrac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta \operatorname{cosec}\theta - 2\sin\theta\cos\theta$

LP : Prove that : $1 + \sec\theta - \tan\theta / 1 + \sec\theta + \tan\theta = 1 - \sin\theta / \cos\theta$

LP : Show that $\sin^6 A + 3\sin^2 A \cos^2 A = 1 - \cos^6 A$

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THANK YOU

COODIES 🥰