[Aim: 100|100 in Maths]

अभय CLASS 10

~~Basics~~

Identities

TRIGONOMETRY

~~L-2~~ L3

sin	cos	tan
P	B	P
H	H	B
cosec	sec	cot

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

$\tan\theta = \frac{1}{\cot\theta} = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

	0	30	45	60	90
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D
$\operatorname{cosec}\theta$	N.D	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	N.D
$\cot\theta$	N.D	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Trigonometric Identities

- $\sin^2\theta + \cos^2\theta = 1$
- $\sec^2\theta - \tan^2\theta = 1$
- $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$\sec^2\theta = 1 + \tan^2\theta$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

LP:

$cosec^2\theta \cdot (1-\cos^2\theta) = k$, find k.

(A) 2

(B) 3

(C) 1

(D) 0

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1-\cos^2\theta$

$$\Rightarrow cosec^2\theta\,\underbrace{(1-\cos^2\theta)}_{\downarrow}$$

$$\Rightarrow \frac{1}{\cancel{\sin^2\theta}} \times \cancel{\sin^2\theta} \Rightarrow 1$$

#L1: $\sin^2\theta + \sin\theta = 1$, find value of $\cos^2\theta + \cos^4\theta$

(A) -1

(B) 1

(C) 0

(D) 2

$$\boxed{\begin{aligned}\sin^2\theta + \cos^2\theta &= 1\\ \sin^2\theta &= 1-\cos^2\theta \checkmark\end{aligned}}$$

$\cancel{1} - \cos^2\theta + \sin\theta = \cancel{1}$

$$\boxed{\cos^2\theta = \sin\theta}$$

$\sin\theta + (\cos^2\theta)^2$

$\Rightarrow \sin\theta + (\sin\theta)^2$

$\therefore \sin\theta + \sin^2\theta$

$\Rightarrow 1 \checkmark$

LP : Prove that : $\frac{\text{Sin A} - 2\,\text{Sin}^3\text{A}}{2\,\text{Cos}^3\text{A} - \text{Cos A}} = \text{Tan A}$

LHS

$$\frac{\sin A\,(1-2\sin^2 A)}{\cos A\,(2\cos^2 A - 1)}$$

$$\Rightarrow \tan A\,\frac{(1-2\sin^2 A)}{(2\cos^2 A-1)}$$

$$\Rightarrow \tan A\,\frac{(1-2(1-\cos^2 A))}{(2\cos^2 A-1)}$$

$\left[\because \sin^2 A+\cos^2 A=1,\ \sin^2 A=1-\cos^2 A\right]$

$$\Rightarrow \tan A\,\frac{(1-2+2\cos^2 A)}{(2\cos^2 A-1)}$$

$$\Rightarrow \tan A\,\frac{(-1+2\cos^2 A)}{(2\cos^2 A-1)}$$

$$\Rightarrow \tan A\,\frac{\cancel{(2\cos^2 A-1)}}{\cancel{(2\cos^2 A-1)}}$$

$$\Rightarrow \boxed{\tan A} = \text{RHS}$$

HP

LP : If $\sin\theta - \cos\theta = 0$, then find the value of $\sin^4\theta + \cos^4\theta$.

$\sin\theta = \cos\theta$

$\boxed{\theta = 45^\circ}$

$\boxed{\sqrt{a} = a^{\frac{1}{2}}}$

$\Rightarrow (\sin 45)^4 + (\cos 45)^4$

$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$

$\Rightarrow 2\left(\frac{1}{\sqrt{2}}\right)^4$

$\Rightarrow 2\,\frac{(1)^4}{(2^{\frac{1}{2}})^{4}} \Rightarrow \cancel{2} \times \frac{1}{2^{\cancel{2}}} \Rightarrow \frac{1}{2}$

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LP : If tan a + cot a = 2 , then tan a^{20} + cot a^{20} =

A. 0
B. 2
C. 20
D. 2^{20}

$\tan\alpha + \frac{1}{\tan\alpha} = 2$

$\Rightarrow \frac{\tan^2\alpha + 1}{\tan\alpha} = 2$

$\Rightarrow \tan^2\alpha + 1 = 2\tan\alpha$

Quadratic

$\boxed{\tan^2\alpha - 2\tan\alpha + 1 = 0}$

$\Rightarrow \tan^2\alpha - \tan\alpha - \tan\alpha + 1 = 0$

$\Rightarrow \tan\alpha(\tan\alpha - 1) - 1(\tan\alpha - 1) = 0$

$\Rightarrow (\tan\alpha - 1)(\tan\alpha - 1) = 0$

$\Rightarrow \tan\alpha - 1 = 0$

$\tan\alpha = 1$

$\cot\alpha = \frac{1}{\tan\alpha} = \frac{1}{1}$

$\therefore \boxed{\cot\alpha = 1}$

$(\tan\alpha)^{20} + (\cot\alpha)^{20}$

$(1)^{20} + (1)^{20}$

$1 + 1 \Rightarrow 2$

#K-3

$\sec^2\theta - \tan^2\theta = 1$

$a^2 - b^2 = (a+b)(a-b)$

$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$$(\sec\theta - \tan\theta) = \frac{1}{(\sec\theta + \tan\theta)} \rightarrow \text{Reciprocal.}$$

→ Can we say the same for $\operatorname{cosec}\theta + \cot\theta$??

we know, $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$a^2 - b^2$

$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

$$\boxed{\operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}}$$

→ can we say same thing for $\sin\theta + \cos\theta$??

$\sin^2\theta + \cos^2\theta = 1$

$a^2 + b^2$

#LP: If $\sec\theta + \tan\theta = 2$

find $\sec\theta - \tan\theta$? !

reciprocals of each other.

$$\sec\theta - \tan\theta = \frac{1}{2}$$

LP : If sec θ + tan θ = p , then tan θ is

A. $p^2 + 1 / 2p$
B. $p^2 - 1 / 2p$
C. $p^2 - 1 / p^2 + 1$
D. $p^2 + 1 / p^2 - 1$

$$\sec\theta - \tan\theta = \frac{1}{p}$$
$$\sec\theta + \tan\theta = p$$
$$-2\tan\theta = \frac{1}{p} - p$$
$$2\tan\theta = p - \frac{1}{p}$$
$$2\tan\theta = \frac{p^2-1}{p}$$
$$\tan\theta = \frac{p^2-1}{2p}$$

LP : Prove the identity : $\dfrac{1}{\text{cosec}\,\theta + \cot\theta} - \dfrac{1}{\sin\theta} = \dfrac{1}{\sin\theta} - \dfrac{1}{\text{cosec}\,\theta - \cot\theta}$

LHS $(\cancel{\text{cosec}\,\theta} - \cot\theta) - \cancel{\text{cosec}\,\theta}$

$\Rightarrow -\cot\theta$

RHS

$\rightarrow \text{cosec}\,\theta - (\text{cosec}\,\theta + \cot\theta)$

$\rightarrow \cancel{\text{cosec}\,\theta} - \cancel{\text{cosec}\,\theta} - \cot\theta$

$\rightarrow -\cot\theta$

LHS = RHS

HP

K^{5}B → addition given है ⟶ multiplication पूछा है

LP : If $\underbrace{\sin\theta + \cos\theta = \sqrt{3}}$, then find the value of $\underbrace{\sin\theta \cdot \cos\theta}$

squaring both sides

sq. both sides

$$\Rightarrow (\sin\theta + \cos\theta)^2 = (\cancel{\sqrt{3}})^{\cancel{2}}$$

$$\Rightarrow \underbrace{\sin^2\theta + \cos^2\theta} + 2\sin\theta\cos\theta = 3$$

$$1 + 2\sin\theta\cos\theta = 3$$

$$\cancel{2}\sin\theta\cos\theta = \cancel{2}$$

$$\sin\theta \cdot \cos\theta = 1$$

LP : If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

$\sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$\sin\theta = \cos\theta(\sqrt{2}-1)$

$\frac{\sin\theta}{\sqrt{2}-1} = \cos\theta$

rationalise

$\Rightarrow \frac{\sin\theta}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \cos\theta$

$\Rightarrow \frac{\sin\theta(\sqrt{2}+1)}{(\sqrt{2})^2-(1)^2} = \cos\theta$

$\Rightarrow \sin\theta(\sqrt{2}+1) = \cos\theta$

$\cancel{2-1} \to 1$

LHS $\cos\theta - \sin\theta$

sin में convert करेंगे

$\sin\theta(\sqrt{2}+1) - \sin\theta$

$\sqrt{2}\sin\theta + \cancel{\sin\theta} - \cancel{\sin\theta}$

$\Rightarrow \sqrt{2}\sin\theta = \text{RHS}$

HP

LP : If $x = p\sec\theta + q\tan\theta$ and $y = p\tan\theta + q\sec\theta$, then prove that $x^2 - y^2 = p^2 - q^2$

$x^2 = (p\sec\theta + q\tan\theta)^2$

$y^2 = (p\tan\theta + q\sec\theta)^2$

To prove : $x^2 - y^2 = p^2 - q^2$

LHS $x^2 - y^2$

$(p\sec\theta + q\tan\theta)^2 - (p\tan\theta + q\sec\theta)^2$

$(a+b)^2$

$\Rightarrow p^2\sec^2\theta + q^2\tan^2\theta + 2(p\sec\theta)(q\tan\theta) - [p^2\tan^2\theta + q^2\sec^2\theta + 2(p\tan\theta)(q\sec\theta)]$

$\Rightarrow p^2\sec^2\theta + q^2\tan^2\theta + \cancel{2pq\sec\theta\tan\theta} - p^2\tan^2\theta - q^2\sec^2\theta - \cancel{2pq\sec\theta\tan\theta}$

$\Rightarrow p^2(\sec^2\theta - \tan^2\theta) + q^2(\tan^2\theta - \sec^2\theta)$

$\Rightarrow p^2(1) - q^2(\underbrace{\sec^2\theta - \tan^2\theta}_{(1)}) \Rightarrow \boxed{p^2 - q^2}$ RHS

HP

LP : Prove that : $\sqrt{1 + \sin A / 1 - \sin A} = \sec A + \tan A$

To prove ⇒ $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Rationalisation

$\sin^2 A + \cos^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$

LHS

$$\sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$$

$$\Rightarrow \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} \Rightarrow \sqrt{\frac{(1+\sin A)^2}{(\cos A)^2}}$$

$$\cancel{\sqrt{\left(\frac{1+\sin A}{\cos A}\right)^2}}$$

$$\Rightarrow \frac{1+\sin A}{\cos A}$$

$$\Rightarrow \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$\Rightarrow \sec A + \tan A = \text{RHS}$$

HP

LP : If $1 + \sin^2\theta = 3 \sin\theta \cos\theta$ then prove that $\tan\theta = 1$ or $\tan\theta = \frac{1}{2}$

$$\Rightarrow \frac{1+\sin^2\theta}{\cos^2\theta} = \frac{3\sin\theta\cancel{\cos\theta}}{\cos^2\theta}$$ (div. both side by $\cos^2\theta$)

$\tan\theta \rightarrow \frac{\sin\theta}{\cos\theta}$

$$\Rightarrow \frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = 3\tan\theta$$

$$\sec^2\theta + \tan^2\theta = 3\tan\theta$$

$$\left(\begin{array}{l} \sec^2\theta - \tan^2\theta = 1 \\ \sec^2\theta = 1 + \tan^2\theta \end{array}\right)$$

$$1 + \tan^2\theta + \tan^2\theta = 3\tan\theta$$

$$1 + 2\tan^2\theta = 3\tan\theta$$

$$\boxed{2\tan^2\theta - 3\tan\theta + 1 = 0}$$

LP: Prove that : $\dfrac{\tan^3\theta}{1+\tan^2\theta} + \dfrac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta \operatorname{cosec}\theta - 2\sin\theta\cos\theta$

LHS $\dfrac{\tan^3\theta}{\sec^2\theta} + \dfrac{\cot^3\theta}{\operatorname{cosec}^2\theta}$

→ सबको sin, cos में change करदो।

$\Rightarrow \dfrac{\frac{\sin^3\theta}{\cos^3\theta}}{\frac{1}{\cos^2\theta}} + \dfrac{\frac{\cos^3\theta}{\sin^3\theta}}{\frac{1}{\sin^2\theta}}$

$\Rightarrow \dfrac{\sin^3\theta}{\cos\theta} + \dfrac{\cos^3\theta}{\sin\theta}$

$\Rightarrow \dfrac{\sin^4\theta + \cos^4\theta}{\sin\theta\cos\theta}$

$\Rightarrow \dfrac{(\sin^2\theta)^2 + (\cos^2\theta)^2}{\sin\theta\cdot\cos\theta}$

$a = \sin^2\theta$

$b = \cos^2\theta$

$\Rightarrow \dfrac{(\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\sin\theta\cdot\cos\theta}$

$\Rightarrow \dfrac{(1)^2 - 2\sin^2\theta\cos^2\theta}{\sin\theta\cdot\cos\theta}$

$\Rightarrow \dfrac{1}{\sin\theta\cdot\cos\theta} - \dfrac{2\sin^2\theta\cos^2\theta}{\sin\theta\cdot\cos\theta}$

$\Rightarrow \operatorname{cosec}\theta\cdot\sec\theta - 2\sin\theta\cdot\cos\theta = $ RHS

$\sec^2\theta - \tan^2\theta = 1$

$\boxed{\sec^2\theta = 1+\tan^2\theta}$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\boxed{\operatorname{cosec}^2\theta = 1+\cot^2\theta}$

$(a+b)^2 = a^2+b^2+2ab$

$\boxed{(a+b)^2 - 2ab = a^2+b^2}$

H.P.

LP : Prove that : 1 + sec θ - tan θ / 1 + sec θ + tan θ = 1 - sin θ / cos θ

#LP: Prove that. $\dfrac{1+(\sec\theta - \tan\theta)}{1+\sec\theta+\tan\theta} = \dfrac{1-\sin\theta}{\cos\theta}$

We know, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \dfrac{\boxed{\underset{a^2 - b^2}{\sec^2\theta - \tan^2\theta}} + (\sec\theta - \tan\theta)}{1+\sec\theta+\tan\theta}$

$\Rightarrow \dfrac{\underset{a+b}{(\sec\theta + \tan\theta)}\underset{a-b}{(\sec\theta - \tan\theta)} + (\sec\theta - \tan\theta)}{1+\sec\theta+\tan\theta}$

$\Rightarrow \dfrac{(\sec\theta - \tan\theta)\left[\cancel{\sec\theta + \tan\theta + 1}\right]}{\cancel{1+\sec\theta+\tan\theta}}$

$\Rightarrow \sec\theta - \tan\theta$

$\Rightarrow \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}$

$\Rightarrow \dfrac{1-\sin\theta}{\cos\theta}$ = RHS

HP

LP : Show that $\sin^6 A + 3\sin^2 A\cos^2 A = 1 - \cos^6 A$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(a+b)^3 - 3ab(a+b) = a^3 + b^3$

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rearrange :-

New LP → $\boxed{\sin^6 A + \cos^6 A = 1 - 3\sin^2 A\cos^2 A}$

LHS

$(\sin^2 A)^3 + (\cos^2 A)^3$

$a^3 + b^3$ $\quad a = \sin^2 A$, $b = \cos^2 A$

$\Rightarrow (a+b)^3 - 3ab(a+b)$

$\Rightarrow (\underbrace{\sin^2 A + \cos^2 A}_{})^3 - 3\sin^2 A\cos^2 A(\underbrace{\sin^2 A + \cos^2 A}_{1})$

$\Rightarrow 1 - 3\sin^2 A\cos^2 A$

$\Rightarrow$ RHS HP

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H/W
15 Question of slides
Ex- 8.3
DPP

THANK YOU

COODIES 🥰

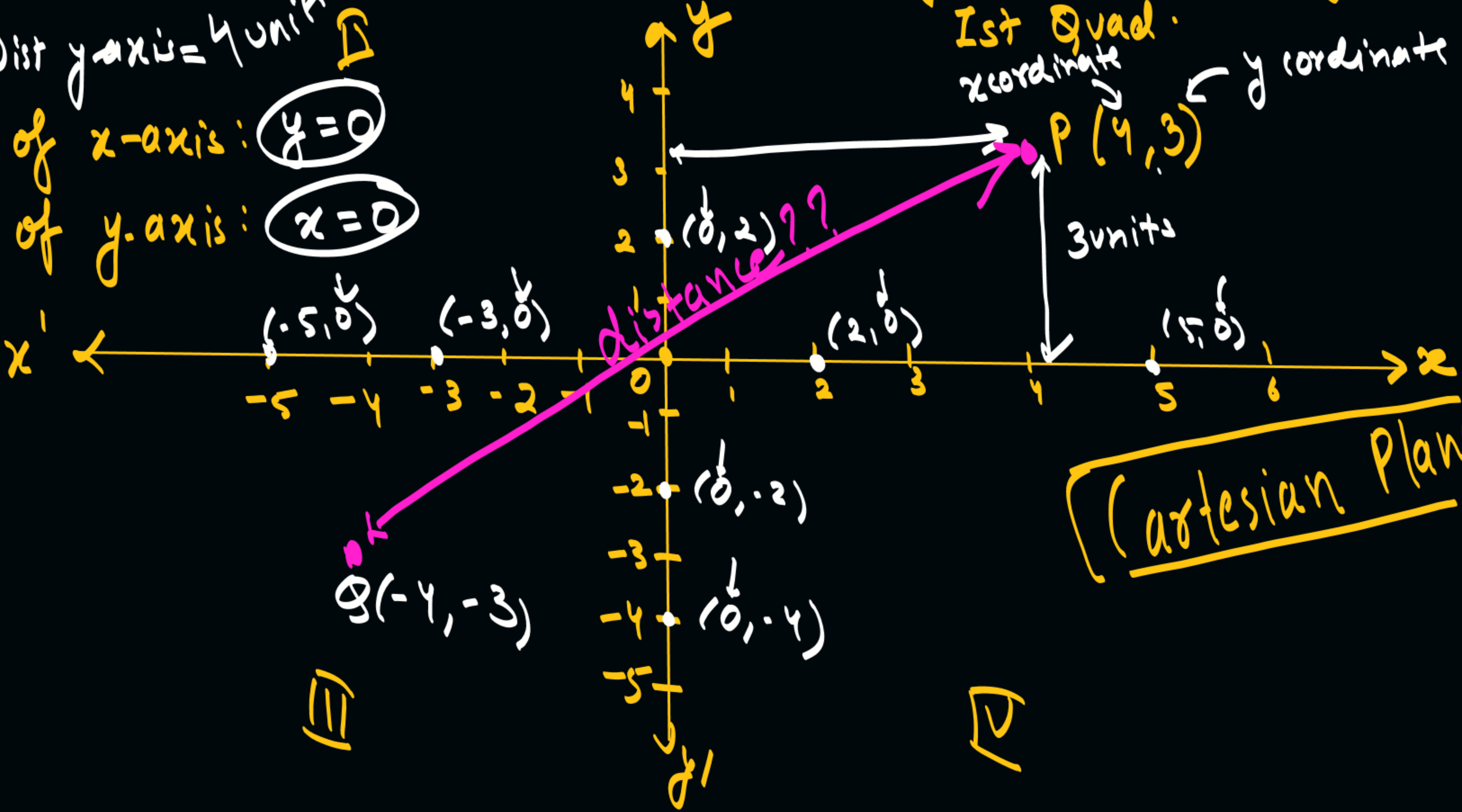
Coordinate Geometry

Q → Dist x axis 3 unit
→ Dist y axis = 4 unit

Dist. of P from x-axis = 3 units
y-axis = 4 unit

Eq^n of x-axis : $y = 0$

Eq^n of y. axis : $x = 0$



(Cartesian Plane)

(I) Distance Formula :-

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$$d = \sqrt{(-4-4)^2 + (-3-3)^2}$$

$$= \sqrt{(-8)^2 + (-6)^2}$$

$$= \sqrt{64 + 36} = \sqrt{100} \Rightarrow 10 \text{ units}$$